# Theory and Practice of Artificial Intelligence

## Other Game Types

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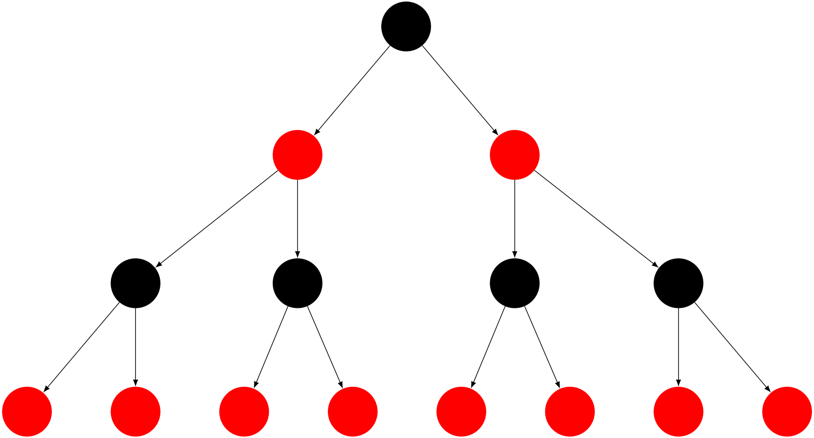
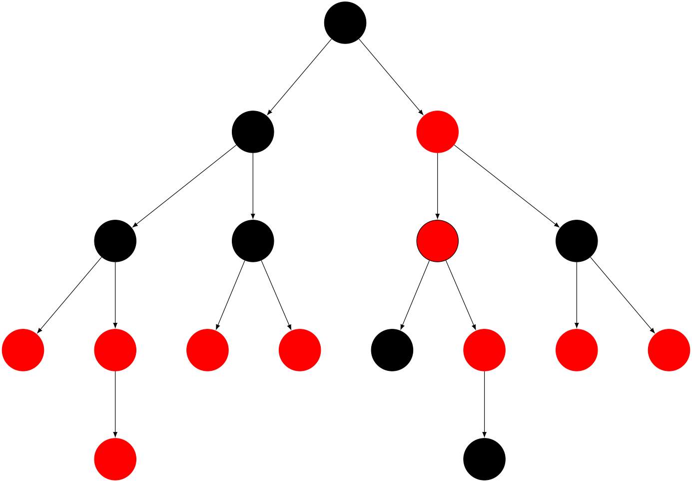
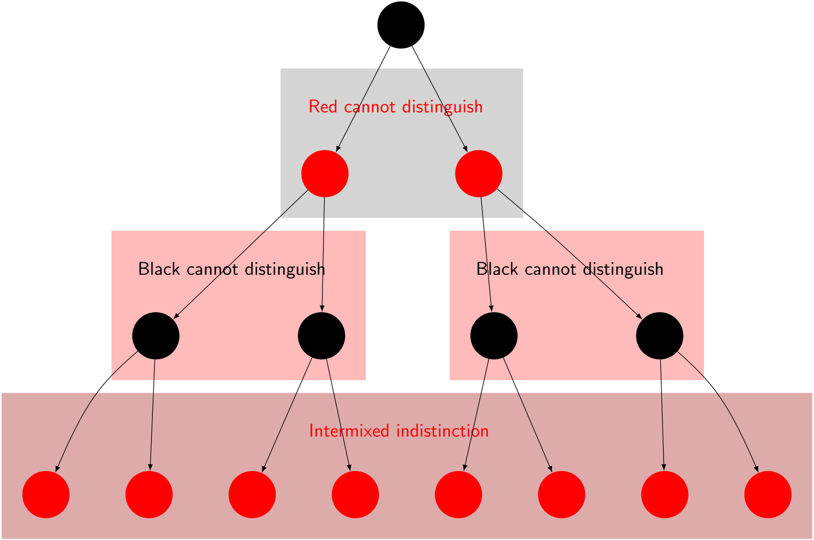
# Game Tree I — Red and Black Alternate



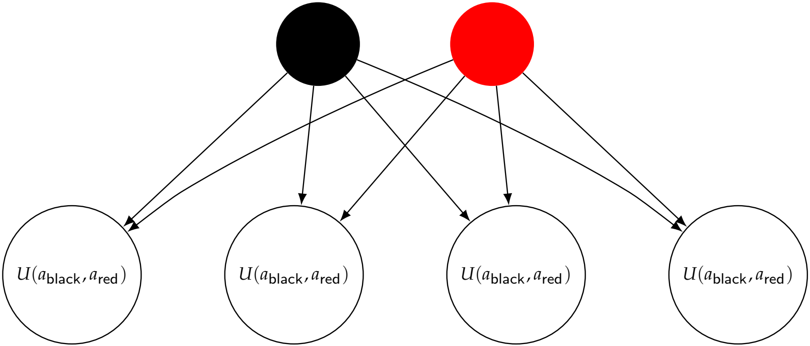
# Game Tree II —
# Red and Black Alternate Irregularly



# Game Tree III — Hidden Info



# Game Tree IV — Simultaneous Moves



# Other Game Types

**Prisoner's Dilemma:** game with

1. simultaneous moves
2. non-zero-sum payoff

| $P1 \backslash P2$ | defect | cooperate |
|---|---|---|
| defect | $-6 \backslash -6$ | $0 \backslash -10$ |
| cooperate | $-10 \backslash 0$ | $-1 \backslash -1$ |

# Dominance

**Def. (strong dominance):** a strategy $s$ for a player $p$ *strongly dominates* $s'$ if the payoff using $s$ is better than using $s'$ for every *fixed* choice of strategy for other players.

**Def. (weak dominance):** a strategy *weakly dominates* if it is better on (at least) one strategy of other players and no worse on any other.

**Def.:** A *dominant strategy* dominates all others.

# Pareto optimality/dominance

**Def. (Pareto optimality):** an outcome is *Pareto optimal* if no other outcome would be preferred by *all* the players.

**Def. (Pareto dominance):** an outcome is strongly *Pareto dominated* if all players would prefer some other outcome

**Def. (weak Pareto dominance):** an outcome is weakly Pareto dominated, if some players would prefer another outcome to which all others would not mind switching

# Dominance in Prisoner's Dilemma

**Note:** both Alice and Bob have a dominant strategy, i.e. we have a dominant strategy *equilibrium*

**Def. (Nash equilibrium):** a selection of strategies for each player such that no player can benefit by switching his/her strategy if all other players' strategies are unchanged.

**Remark:** the *dilemma* in the prisoner's dilemma is due to the fact that the Nash equilibrium $(-6, -6)$ of both prisoners defecting is Pareto dominated by $(-1, -1)$ of both prisoners cooperating.

**Note:** a Nash equilibrium can arise even without the existence of a dominant strategy.

# Comments on Game Theory

**Remark:** if

- the prisoner's dilemma game is being iterated
- the players are allowed to have memories and identify their opponent

this can lead to solutions which avoid the equilibrium.

**Note:** Tit-For-Tat and very related strategies prove to be remarkably stable and robust solutions.

**Remark:** if one has a Pareto-optimal point which is also a Nash equilibrium, then we call that a *solution* of the game.

**Consider:** simultaneous zero-sum games. Need to consider only the payoff $P$ for one of the players, the other will follow as $-P$.

**2-Finger Morra:** payoff matrix:

| $E \backslash O$ | 1 | 2 |
|---|---|---|
| 1 | $2 \backslash -2$ | $-3 \backslash 3$ |
| 2 | $-3 \backslash 3$ | $4 \backslash -4$ |

**Goal:** find *solution*

# Zero-Sum Games: Solution

**Scenario 1:** force $E$ to begin, $O$ to follow. This is an advantage for $O$. Thus, $E$ is guaranteed an outcome of $U_E \geq -3$.

**Scenario 2:** force $O$ to begin, $E$ to follow. $O$ can ensure an outcome with $U_E \leq 2$.

# Mixed Strategy

**Note:** revealing a strategy gives the second player an advantage.
For, if second player plays $[p : 1; (1-p) : 2]$ (notation: lottery where outcome 1 is selected with probability $p$ and outcome 2 is selected with probability $1-p$), the expected utility for $E$ is

$$pU_E(O = 1) + (1-p)U_E(O = 2)$$

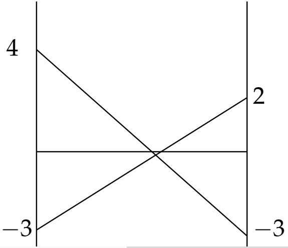
If $U_E(O = 1)$ and $U_E(O = 2)$ are different, $O$ should pick the best as *pure* strategy.

# Utilities for Mixed Strategies I

**Assume:** $E$ moves first, $O$ does not know the move, but knows $p$ in $E$'s strategy $[p:1;(1-p):2]$. Then, if

1. $O$ chooses 1, then
   $\mathbf{E}(U) = 2p - 3(1-p) = 5p - 3$
2. $O$ chooses 2, then
   $\mathbf{E}(U) = -3p + 4(1-p) = 4 - 7p$ .

**Thus:**
- $O$ will always pick the minimum of both
- $E$ will pick $p$ such that this minimum is maximal
- i.e. resulting payoff is $U = -\frac{1}{12}$.



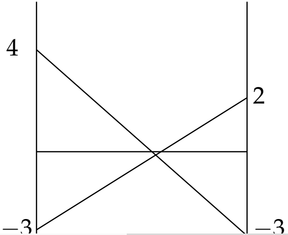
# Utilities for Mixed Strategies II

**Assume:** $O$ moves first, probabilites $[q : 1; (1-q) : 2]$. If

1. $E$ picks 1, then $\mathbf{E}(U) = 2q - 3(1-q) = 5q - 3$
2. $E$ picks 2, then $\mathbf{E}(U) = -3q + 4(1-q) = 4 - 7q$

**Thus:**
- $E$ picks the maximum of both
- $O$ picks $q$ such that this maximum is minimal
- i.e. value becomes $U = -\frac{1}{12}$ .

**Note:** The two $U$ values enclose the true value, which is therefore $U = -\frac{1}{12}$. It turns out that $p = \frac{7}{12} = q$.



# Minimax Equilibria

**Bottom Line:** there exists an *equilibrium*, a *minimax* equilibrium which is Nash equilibrium.

**von Neumann:** every two-player zero-sum game has a minimax equilibrium on mixed strategies. Also, in zero-sum games, Nash equilibria are minimax equilibria.